Beckham Carver

Homework 9

STAT 4025/5025 – Due Sunday, April 3rd 11:00 pm 32 points total for both 4025 and 5025

1. The file *ortho\_study.csv* contains data from a published medical journal article on the study of the use of bone anchors for the repair of torn rotator cuffs (a common shoulder injury). The bone anchor is a threaded insert that is screwed into a hole that has been drilled into the shoulder bone near the site of the torn tendon. The torn tendon is then sutured to the anchor. In a successful procedure, the tendon is stabilized and reattaches itself to the bone. However, if the repair is subject to high loads, the bone anchor can pull out. An experiment was performed to study the force required to pull out the anchor for three types of anchors and two different foam densities (foam simulates the natural ability found in human bone). Two replicates were performed. Experimental units were shoulders from human cadavers.
   1. Using a one-way ANOVA, determine if there is a significant difference among the anchor types – state your hypotheses (1 pt) and make a conclusion based upon a p-value (1 pt) as well as a graphical presentation of 95% confidence intervals on the group means using the plotmeans() function (2 pts). [4 total points].

H0: μa = μb = μc

H1: At least one of the means is different

Chart

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With a p-value greater than 0.05, we accept null hypothesis that there is no mean difference among the anchor types alone.

* 1. By hand, construct the 95% confidence interval that R used for Anchor Type C in part a. [2 pts]

Sample mean = (210 + 205 + 322 + 340) / 4 = 269.25

Sample SD = √ [ (210 – 269.25)2 + ... + (340 – 269.25)2/ 4 ] = 71.70948

Sample size = 4

T value = 3.182446

= 269.25 – 3.182446 \* ( 71.70948 / √4 ) < μ

< 269.25 + 3.182446 \* ( 71.7094 / √4 )

= 269.25 – 3.182446 \* ( 71.70948 / 2 ) < μ

< 269.25 + 3.182446 \* ( 71.7094 / 2 )

= 155.1442 < μ < 383.3556

* 1. For the one-factor setting considered in part a, what is the estimate of experimental error variation? What are some obvious contributors to this variance? [3 pts]

Our experimental error variation (MSE) is equal to 1623.479

What is likely contributing to this is that the one-factor setting is not accounting for the effect of foam density on the model, which is being accounted for in our high MSE.

* 1. Conduct the multi-factor ANOVA for this data – factors are Anchor Type and Foam Density…include the interaction term. State your hypotheses and make conclusions about each set of hypotheses (6 pts).

Where anchor type is “i” and foam density is “j”

yijk = μ + αi+ βj + (αβ)ij + εijk

H0: μ = μAncℎ = μFoam = μ(AnchFoam)

H1: μ ≠ μAncℎ, μFoam, or μ(AnchFoam)

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Here we see all 3 p-values are below 0.05, this means that we can conclude there is a true difference in means for anchor type and foam density. We also conclude that there is a significant interaction between the two.

* 1. For the multi-factor setting in part d, provide the estimate for the experimental error variation – interpret this quantity. Which estimate of variance is larger between the one you provided in part c and the one here? Provide an explanation for the difference in these two estimates of experimental error variation. [5 pts – 2 pts for estimate and 3 pts for explanation]

Our experimental error variation (MSE) for the multi-factor model is equal to 38.29167

This is significantly less than our original model MSE. By not accounting for the foam density and the interaction, our previous model had been swung so far in the wrong direction that our p-value led to a conclusion of no mean difference. Our new model now concludes that there is a true mean difference.

The explanation of these separate conclusions from the same data lies in the MSE’s. The first model essentially ‘lost’ the correct conclusion within its high MSE, due to high residuals. By accounting for foam density, and the interaction, we have reduced our residuals, and in turn our MSE, and ‘uncovered’ the correct conclusion.

* 1. Produce and interpret an interaction plot between Anchor Type and Foam Density. (3 pts)

Chart, line chart

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This interaction plot shows a distinct interaction between high foam density and anchor type, and a relative level of interaction between low density and anchor type. This difference in change between the high and low density shows that the affect of anchor type is highly dependent on the foam density, signifying interaction.

* 1. Using Tukey’s HSD, find a 95% confidence interval on the difference in mean pull strength for Anchor Type B with high foam density and Anchor Type B low foam density – do this in R AND by hand. [4 pts]

Text

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Groups = 2

d.F. = 2

Critical value = 6.09

Low mean = (230 + 237) / 2 = 233.5

High mean = (195 + 210) / 2 = 202.5

HSD = 6.09 \* sqrt(68.5/4)

HSD = 25.20186

* 1. Using the contrast approach, find the 95% confidence interval on the difference in mean pull strength for Anchor Type B with high foam density and Anchor Type B low foam density – do this in R (1 pt) AND by hand (2 pts). Show which betas refer to each mean (2 pts) [5 pts]

R code:

library(tidyverse)

library(gplots)

library(multcomp)

#read file

anchor <- read.csv("ortho\_study.csv")

# one way anova, one factor

m1 <- lm(pull\_force~Anchor\_Type, data = anchor)

anchor$resids <-residuals(m1)

anova(m1)

# CI plot

plotmeans(pull\_force~Anchor\_Type,xlab="Anchor Type",ylab="Pull Force", p=.95,

main="Ortho Main Effect Plot",barcol="black",

n.label=F,data=anchor)

# Calculate CI terms

c\_anchor <- filter(anchor, Anchor\_Type == "C")

mean(c\_anchor$pull\_force)

sd(c\_anchor$pull\_force)

qt(p = 0.05/2, df = 4, lower.tail = F)

# calculate MSE

mean(anchor$resids^2)

# multi-factor anova

m2 <- lm(pull\_force~Anchor\_Type + Foam\_Dense + Anchor\_Type:Foam\_Dense, data = anchor)

anova(m2)

anchor$resids\_multi <- residuals(m2)

# calculate new MSE\

mean(anchor$resids\_multi^2)

# interaction plot

interaction.plot(anchor$Anchor\_Type, anchor$Foam\_Dense, anchor$pull\_force,

fun = mean,

type = "b",

pch=c(1:3),

legend = TRUE,

trace.label = "Foam Density",

fixed = FALSE,

xlab = "Anchor Type",

ylab = "Pull force")

# Tukey for type B

b\_anchor <- filter(anchor, Anchor\_Type == "B")

m3 <- aov(pull\_force~Foam\_Dense, data = b\_anchor)

summary(m3)

TukeyHSD(m3, conf.level = .95)

qt(p=0.05, df = 4, lower.tail = F)

# Contrast

???????